International journal of Engineering Research
&
Management Technology

ISSN:2348-4039
Volume-1,Issue-1
January 2013

Production Inventory Model with Multi-Variate Demand and Partial Backlogging

Anand

Research Scholar

Singhania University,

Rajasthan

Dr.Jagat Veer Singh

Head of Department

All India Jat Heroes Memorial College

Rohatak (Haryana)

ABSTRACT:

A production inventory model for decaying items with multi-variate demand and variable holding cost has been developed. Demand rate function depends upon the present inventory level and the selling price per unit during production phase, then during deterioration, it is a decreasing quadratic function and during backlogging, it is a constant. Shortages are permitted with partial backlogging. The backlogging rate is waiting time for the next replenishment. Finally, numerical examples are presented to demonstrate the developed model and solution procedure. The sensitivity analysis is applied on the effect of the major parameters.

KEY WORDS: Multi-Variate Demand, Partial Backlogging, Shortages cost

1. INTRODUCTION

A new concept, but a common sense conclusion. It is a general observation that a In the classical inventory models, the demand rate is regularly assumed to be either constant or time-dependent or stock dependent but independent of the selling price. If the goal is to minimize the cost, then a stock-dependent demand rate causes a lower level of inventory than the traditional EOQ model with a constant demand rate. It defeats the purpose of a stock-dependent demand rate that is desirable to maintain large inventory for potential profits obtained from the increased demand.

The dependence of the sale of any item on its selling price is not n increase in the selling price of the commodity will deter its customer's from opting that item in future. However, a dip in the selling price, in whatever form it may come, always notices a sudden increase in the demand rate, as a reduction in prices encourages the customers to buy more.

In many real-life situations, the practical experiences reveal that some but not all customers will wait for backlogged items during a shortage period, such as for fashionable commodities or high-tech products with short product life cycle. The longer the waiting time is, the smaller the backlogging rate would be. According to such phenomenon, taking the backlogging rate into account is necessary. However, most of the inventory models unrealistically assume that during stock out either all demand is backlogged or all is lost. In reality often some customers are willing to wait until replenishment, especially if the wait will be short, while others are more impatient and go elsewhere. The backlogging rate depends on the time to replenishment-the longer customers must wait, the greater the fraction of lost sales.

2. ASSUMPTIONS AND NOTATIONS

Assumptions:

1. The demand rate is a known function and varies during the ordering cycle in the time interval as shown below:

$$D(t)=\begin{cases}\alpha(p)+\beta I(t), 0<t<t_1\\ at^2+b, t_1<t<t_2\\ D, t_2<t<T\end{cases} \qquad \ldots(1)$$

Where $\alpha(p)$ is a non-negative function of the selling price 'p' and β, a, b, D are non-negative constants.

2. To make the production cycle repeatable, we assume that the initial and ending inventory level during the production cycle are same, i.e. Q, (Q>0).
3. The replenishment occurs immediately at a very large rate.
4. The deterioration rate is constant and there is no replacement of the deteriorated items.
5. Holding cost is linear increasing function of time.
6. Shortages are allowed with partial backlogging.
7. The backlogging rate is waiting time for the next replenishment.

Notations:

θ	The constant deterioration rate, $0<\theta<1$
t_1	The productions run time (decision variable)
t_2	The time at which shortages starts
T	The length of ordering cycle (it includes the backlogging duration after the production cycle)
K	The production rate
I(t)	The inventory level at time t, Q<I(t)<P
P	The maximum inventory level during the production cycle
$\frac{1}{1+\delta(T-t)}$	Backlogging rate
$C_h+\gamma t$	Holding cost per unit per unit time
C_d	Deterioration cost per unit per unit time
C_S	Shortages cost per unit per unit time
C_{LS}	Lost sale cost per unit per unit time

3.MATHEMATICAL MODEL

With an initial inventory level 'Q', the production starts at t = 0 till $t = t_1$ when the maximum production level 'P' is reached. Here the production is stopped and the inventory gradually deteriorates to 'Q' at $t = t_2$. Then the inventory level further depletes to zero due to partial backlogging. The objective of the inventory is to optimize the length of the cycle and the shortages point to minimize the total cost. Here shortages are allowed to occur during the time interval $[t_2, T]$. During this interval, all the demands are partially backlogged. The graphical representation of the inventory system is shown in figure 1. During inventory interval $[0, t_1]$ a production rate starts where the inventory level attains its maximum value P. Hence the differential equation representing the inventory is:

$$\frac{dI(t)}{dt} + \theta I(t)dt = K - D(I(t), p) \qquad 0 \le t \le t_1 \qquad \ldots(2)$$

Where I(0) = Q

During deterioration phase of the inventory, i.e. in interval $[t_1, t_2]$, the inventory level decreases due to constant deterioration as well as the demand rate. Hence the differential equation is represented as:

$$\frac{dI(t)}{dt} + \theta I(t)dt = -D(t) \qquad t_1 \le t \le t_2 \qquad \ldots(3)$$

Where $I(t_2) = Q$

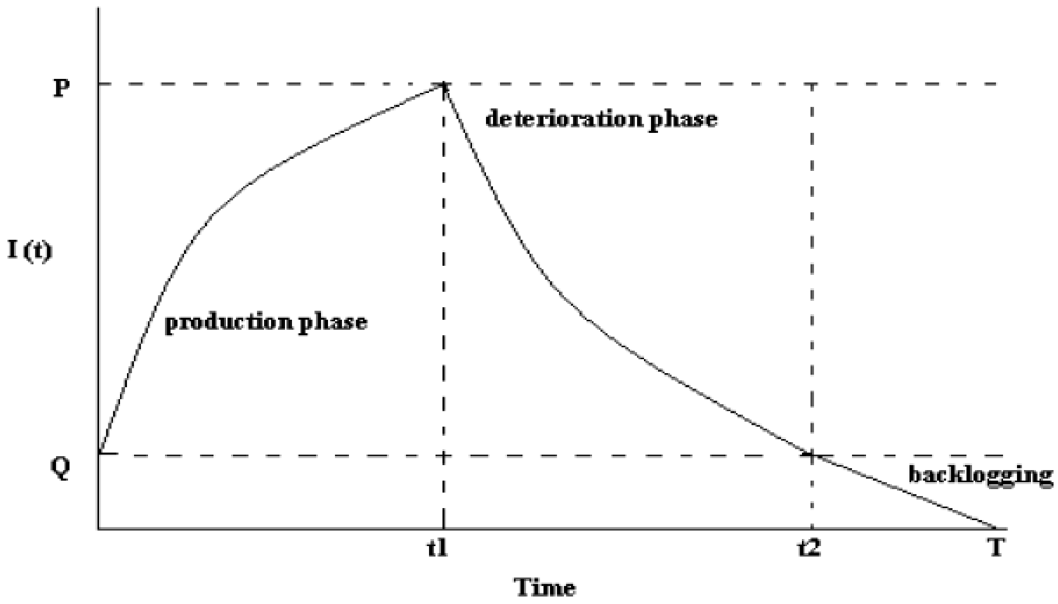

Fig 1: Graphical representation of the Mathematical Model

During shortages interval [t_2,T] of the inventory, the demand rate at time *t* is partially backlogged by a fraction $\frac{1}{1+\delta(T-t)}$. Thus the differential equation representing the amount of demand backlogged is:

$$\frac{dI(t)}{dt} = -\frac{D}{1+\delta(T-t)} \qquad t_2 \le t \le T \qquad \ldots(4)$$

Where $I(t_2) = Q, I(T) = 0$

We solve these equations and the calculate the different costs per cycle viz. holding cost, deterioration cost, shortages cost, opportunity cost and finally minimize the average total cost per unit cycle.

Solving the equation (2) using equation (1) and the boundary conditions, we get

$$I(t) = \frac{(K-\alpha(p))}{\theta+\beta}[1-e^{-(\theta+\beta)t}] + Qe^{-(\theta+\beta)t} \qquad 0 \le t \le t_1 \qquad \ldots(5)$$

Solving the equation (3) for $t_1 \le t \le t_2$

$$\frac{dI(t)}{dt}+\theta I(t)dt=-(a(t-t_1)^2+b) \qquad t_1\le t\le t_2 \qquad \ldots(6)$$

Solving the differential equation and substituting the boundary condition $I(t_2)=Q$, we get

$$I(t)=Q+\frac{at_2^{\,2}-at^2}{\theta}+\frac{2at-2at_2}{\theta} \qquad \ldots(7)$$

Similarly solving equation (4) using equation (1) and the boundary conditions, we get

$$I(t)=\frac{D}{\delta}\log[\frac{1+\delta(T-t+t_2)}{1+\delta(t_2)}] \qquad \ldots(8)$$

Here, we have $1+\delta(t_2)>0$. From Equation (5) and equation (7), hence,

$$\frac{(K-\alpha(p))}{\theta+\beta}[1-e^{-(\theta+\beta)t_1}]+Qe^{-(\theta+\beta)t_1}=Q+\frac{at_2^{\,2}-at_1^{\,2}}{\theta}+\frac{2at_1-2at_2}{\theta} \qquad \ldots(9)$$

From equation (7) and equation (8), we have $I(t_2)=Q$, i.e.

$$T=\frac{e^{Q\delta/D}[1+\delta(t_2)]-1}{\delta} \qquad \ldots(10)$$

We get the maximum amount of demand backlogged per cycle

$$S=I(t_2)=Q \qquad ...(11)$$

Inventory holding cost per cycle is as follows:

$$H.C.=[\int_0^{t_1}(C_h+\gamma t)I(t)dt+\int_{t_1}^{t_2}(C_h+\gamma t)I(t)dt] \qquad \ldots(12)$$

Deterioration cost per cycle:

$$D.C.=C_d[P-\int_0^{t_2}D(t)dt]$$

$$=C_d[Q+\frac{at_2^{\,2}-at_1^{\,2}}{\theta}+\frac{2at_1-2at_2}{\theta}-\int_0^{t_1}D(t)dt-\int_{t_1}^{t_2}D(t)dt] \qquad \ldots(13)$$

Shortages cost per cycle

$$S.C.=-C_S[\int_{t_2}^{T} I(t)dt] \qquad \ldots(14)$$

Opportunity cost due to lost sale per cycle is

$$S.C.=C_{LS}[\int_{t_2}^{T}(1-\frac{1}{1+\delta(T-t+t_2)})Ddt] \qquad \ldots(15)$$

Hence total average cost per unit cycle is $TC_{avg}=TC_{avg}(t_1, t_2, T)$

$$T.C.=\frac{1}{T}[H.C.+D.C.+S.C.+L.S.C.] \qquad \ldots(16)$$

From equations (12), (13), (14) and (15), the total cost is as follows:

$$T.C.=\frac{1}{T}[\int_{0}^{t_1}(C_h+\gamma t)I(t)dt+\int_{t_1}^{t_2}(C_h+\gamma t)I(t)dt]$$

$$+C_d[Q+\frac{at_2^{\,2}-at_1^{\,2}}{\theta}+\frac{2at_1-2at_2}{\theta}-\int_{0}^{t_1}D(t)dt-\int_{t_1}^{t_2}D(t)dt]-C_S[\int_{t_2}^{T}I(t)dt]$$

$$\left.+C_{LS}[\int_{t_2}^{T}(1-\frac{1}{1+\delta(T-t+t_2)})Ddt]\right] \qquad \ldots(17)$$

$$T.C.=\frac{1}{T}\left[(C_h-C_d)\{\frac{\alpha(p)(\theta-1)+K(\beta+1)}{\theta+\beta}t_1+\frac{K-\alpha(p)-Q(\theta+\beta)}{\theta+\beta}(e^{-(\theta+\beta)t_1}-1)]\right.$$

$$+(C_h-C_d)][t_2^{\,2}(\frac{2a}{3\theta}-\frac{a}{3})+(t_1^{\,2}+t_2^{\,2})(-\frac{a}{\theta^2})+(t_2-t_1)(Q-b)$$

$$+t_1^{\,3}(\frac{a}{3\theta}+\frac{a}{3})+t_1(-\frac{at_2^{\,2}}{\theta}+\frac{2at_2}{\theta^2})]+C_d[Q+\frac{at_2^{\,2}-at_1^{\,2}}{\theta}+\frac{2at_1-2at_2}{\theta}]$$

$$+\gamma[(\frac{K-\alpha(p)}{\theta+\beta})(\frac{t_1^{\,2}}{2}+\frac{(1-Q)t_1e^{-(\theta+\beta)t_1}}{(\theta+\beta)}+\frac{(e^{-(\theta+\beta)t_1}-1)}{(\theta+\beta)^2}+\frac{Q(e^{-(\theta+\beta)t_1}+1)}{(\theta+\beta)^2})]$$

$$+\gamma[(\frac{Qt_2^{\,2}}{2}-\frac{Qt_1^{\,2}}{2}-\frac{at_1^{\,2}t_2^{\,2}}{2\theta}+\frac{at_2^{\,4}}{4\theta}+\frac{at_1^{\,4}}{4\theta}+\frac{2at_2^{\,3}}{3\theta}-\frac{2at_1^{\,3}}{3\theta}-\frac{at_2^{\,3}}{\theta}+\frac{at_1^{\,2}t_2}{\theta}]$$

$$-C_S[-\frac{D(1+\delta t_2)\log(1+\delta t_2)-D(1+\delta T)\log(1+\delta T)}{\delta^2}-\frac{D\log(1+\delta t_2)(T-t_2)}{\delta}]$$

$$+C_{LS}[D(T-t_2)+\frac{D}{\delta}\log\frac{t_2}{T}]\Big] \qquad \ldots(18)$$

4. Numerical Illustrations:

The following numerical data has been used to find the optimal solution of the total cost and other parameters. The values of the parameters are:

α = 46, β =3.4, K=165, θ=0.05, δ=0.03, γ=0.05, C_h=1.6, C_d=1.4, C_O=0.6, C_S=0.8, C=1.2, a=1.4, b=2.6, Q=17, D=15, N=200, G=5, H=0.04

Using the above mentioned parameter values, the optimal production length t_1=0.1641 unit time, the optimal shortages point t_2=0.7283 unit time and the optimal length of the ordering cycle T=2.6574 unit time. The minimum average total cost per unit time T.C.=20.4213

5. CONCLUSION

An EPQ inventory model for decaying items with multi variate demand has been developed. The demand rate depends upon the present inventory level and selling price per unit and it is a decreasing quadratic function during deterioration and constant during shortages. Shortages are allowed with partial backlogging. Backlogging rate is inversely proportional to the waiting time for the next replenishment. Holding cost is linear increasing function of time. Finally numerical example and sensitivity analysis are given to validate the results of the production-inventory model. The system is solved using the mathematical software ***MATHMATICA 5.2***. This way, the whole research caters to put forward some aspects of an inventory model with some real world considerations and market deliberations. Therefore, the presented mathematical model is much more realistic and practical.

6. **REFERENCES**

[1]. **Abad, P.L. (1996).** "Optimal pricing and lot sizing under conditions of perishability and partial backordering", *Management Science; 42: pp. 1093-1104.*

[2]. **Abad, P.L. (2000).** "Optimal lot size for a perishable good under conditions of finite production and partial backordering and lost sale". *Computers & Industrial Engineering; 38(4): pp. 457-465.*

[3]. **Abad, P.L., (2008),** "Optimal price and order size under partial backordering incorporating shortage, backorder and lost sale costs", *Int. J. Production Economics 114 179–186.*

[4]. **Covert, R.P., Philip, G.P. (1973).** "An EOQ model for items with Weibull distribution deterioration", AIIE Trans; 5(4): 323-329.

[5]. **Ghare, P.M., Schrader, G.F. (1963).** "A model for exponentially decaying inventory", Journal of the Industrial Engineering; 14: 238-243.

[6]. **Goyal, S.K., Chang, C.T., (2009),** "Optimal ordering and transfer policy for an inventory with stock dependent demand", *European Journal of Operational Research, 196,177-185.*

[7]. **Hsieh, T.P., Dye, C.Y., Ouyang, L.Y., (2010),** "Optimal lot size for an item with partial backlogging rate when demand is stimulated by inventory above a certain stock level", *Mathematical and Computer Modelling, 51, 13-32.*

[8]. **Jolai, F., Tavakkoli-Moghaddam, R., Rabbani, M., Sadoughian, M.R. (2006).** "An economic production lot size model with deteriorating items, stock-dependent demand, inflation, and partial backlogging", *Applied Mathematics and Computation; 181: pp. 380-389.*

[9]. **Khanra, S., Chaudhuri, K.S. (2003).** "A note on an order-level inventory model for a deteriorating item with time-dependent quadratic demand". *Computers and Operations Research; 30: 1901-1916.*

[10]. **M. Khouja, R.L. Kumar (2002). "**Information technology investments and volume-flexibility in production systems", *International Journal of Production Research, 40(1), pp. 205-223.*

[11]. **Ouyang, L.Y., Wu, K.S., Cheng, M.C. (2005). "**An inventory model for deteriorating items with exponential declining demand and partial backlogging", *Yugoslav Journal of Operations Research; 15(2): pp. 277-288.*

[12]. **Papachristos, S., Skouri, K. (2000).** "An optimal replenishment policy for deteriorating items with time-varying demand and partial-exponential type backlogging", *Operations Research Letters; 27(4): pp. 175-184.*

[13]. **Bansal, K.K.,. (2013), "**Inventory Model for Deteriorating Items with the Effect of inflation", International Journal of Application or Innovation in Engineering & Management, Volume 2, Issue 5,